Level – 1	Daily Tutorial Sheet - 5

61.(A) $\displaystyle\int \frac{\log(x+1)-\log x}{x(x+1)}\,dx$

$$\int (\log(x+1)-\log x)\,.\,\frac{1}{-x(x+1)}\,dx = \frac{-1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2 + C$$

$$\left[\frac{d}{dx}\left[(x+1)-\log x\right] = \frac{1}{x+1}-\frac{1}{x} = -\frac{1}{(x+1)\,x}\right]$$

62.(A) $\displaystyle\int \frac{dx}{5+4\sin x} = \int \frac{dx}{5+4\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$

$$= \int \frac{\sec^2\frac{x}{2}}{5\tan^2\frac{x}{2}+8\tan\frac{x}{2}+5} = 2\int \frac{dz}{5z^2+8z+5} \qquad \left[\text{Putting } \tan\frac{x}{2} = z \ \Rightarrow\ \sec^2\frac{x}{2}\,dx = 2\,dz\right]$$

$$= \frac{2}{5}\int \frac{dz}{z^2+1+\frac{8}{5}z} = \frac{2}{5}\int \frac{dz}{\left(z+\frac{4}{5}\right)+\left(\frac{3}{5}\right)^2} = \frac{2}{3}\cdot\frac{5}{3}\tan^{-1}\left(\frac{z+\frac{4}{5}}{\frac{3}{5}}\right)+C = \frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}+4}{3}\right)+C$$

$\therefore \quad A = \dfrac{2}{3}$ and $B = \dfrac{5}{3}$

63.(B) Let $I = \displaystyle\int x\,\frac{e^x}{\sqrt{1+e^x}}\,dx$

Putting $1+e^x = t^2$ *i.e.* $x = \log(t^2-1)$, we get:

$$I = 2\int \log(t^2-1)\,dt = 2\left[t\log(t^2-1)-2\int \frac{t^2}{t^2-1}\,dt\right] = 2\left[t\log(t^2-1)-2\int\left\{1+\left(\frac{1}{t^2-1}\right)\right\}dt\right]$$

$$= 2\left[t\log(t^2-1)-2t-\log\left(\frac{t-1}{t+1}\right)\right]+C = 2x\sqrt{1+e^x}-4\sqrt{1+e^x}-2\log\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)+C$$

Hence, $f(x) = 2x-4 = 2(x-2)$ and $g(x) = \dfrac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}$

64.(C) $\displaystyle I = \int \frac{dx}{5+4\cos x} = \int \frac{dx}{5\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)+4\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)} = \int \frac{dx}{9\cos^2\frac{x}{2}+\sin^2\frac{x}{2}} = \int \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}\,dx$

Let $t = \tan\dfrac{x}{2} \Rightarrow 2dt = \sec^2\dfrac{x}{2}\,dx$

$$\Rightarrow \quad I=\int \frac{2dt}{9+t^2}=\frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right)+C=\frac{2}{3}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{3}\right)+C \Rightarrow k=\frac{2}{3}, m=\frac{1}{3}$$

65.(B) $I=\int \frac{1}{(x+5)\sqrt{x+4}}dx$. Put $x+4=t^2$ to get $dx=2t\,dt$

$$\Rightarrow \quad I=\int \frac{2t\,dt}{\left(t^2+1\right)t}=2\int \frac{dt}{t^2+1}=2\tan^{-1}(t)+c=2\tan^{-1}\left(\sqrt{x+4}\right)+c$$

66.(D) $\int\left[e^x \sec x+e^x \,.\, \log(\sec x+\tan x)\right]dx=\int e^x\left[\log(\sec x+\tan x)+\sec x\right]dx$

$$=\int e^x\left[\log(\sec x+\tan x)+\frac{d}{dx}\left[\log(\sec x+\tan x)\right]\right]dx=e^x \,.\, \log(\sec x+\tan x)+c$$

$$=-e^x \,.\, \log(\sec x-\tan x)+c$$

67.(B) Put $x^2=t$ and use partial fractions.

68.(B) $\int \frac{1}{x}\log_{ex} e \,\log_{e^2x} e\,dx=\int \frac{1}{x}\left(\frac{\log e}{\log e+\log x}\right)\left(\frac{\log e}{2\log e+\log x}\right)dx=\int \frac{1}{x}\frac{1}{(1+\log x)}\frac{1}{(2+\log x)}dx$

Let $\log x=k \quad \Rightarrow \quad \int \frac{1}{e^k}\frac{1}{(1+k)}\frac{1}{(2+k)}e^k\,dk=\int\left(\frac{1}{1+k}-\frac{1}{2+k}\right)dk=\log\left(\frac{1+k}{2+k}\right)$

$\Rightarrow \quad f(x)=k=\log x \quad \Rightarrow \quad \lim_{x\to 0}\frac{\log(x+1)}{x}\left(\frac{0}{0}\text{ form}\right) \quad \Rightarrow$ from L.H. rule $\lim_{x\to 0}\frac{1}{x+1}=1$

69.(C) $\int e^x(\tan x-\log\cos x)=-e^x\log\cos x+c=e^x\log\sec x+c \Rightarrow \quad e^x$ has range R^+.

70.(D) $\int \frac{1}{(\sin x+4)(\sin x-1)}dx=\frac{1}{5}\int \frac{(\sin x+4)-(\sin x-1)}{(\sin x+4)(\sin x-1)}dx=\frac{1}{5}\int \frac{1}{\sin x-1}dx-\frac{1}{5}\int \frac{1}{\sin x+4}dx$

$$=\frac{1}{5}\int \frac{2dt}{2t-1-t^2}-\frac{1}{5}\int \frac{2dt}{2t+4\left(1+t^2\right)} \qquad \left[\text{Putting } \tan\frac{x}{2}=t\right]$$

$$=-\frac{2}{5}\int \frac{dt}{t^2-2t+1}-\frac{1}{10}\int \frac{dt}{t^2+\frac{1}{2}t+1}=-\frac{2}{5}\int \frac{1}{(t-1)^2}dt-\frac{1}{10}\int \frac{1}{\left(t+\frac{1}{4}\right)^2+\left(\frac{\sqrt{15}}{4}\right)^2}dt$$

$$=\frac{2}{5}\frac{1}{(t-1)}-\frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right)+C=\frac{2}{5}\cdot\frac{1}{\tan\frac{x}{2}-1}-\frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}\right)+C$$

$$\therefore \quad A=\frac{2}{5}, B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$$

71.(C) $\int\left[f(x)g''(x)-f''(x)g(x)\right]dx=\int f(x)\,g''(x)dx-\int f''(x)g(x)\,dx$

$$=\left(f(x)\,g'(x)-\int f'(x)g'(x)dx\right)-\left(g(x)\,f'(x)-\int g'(x)\,f'(x)dx\right)=f(x)\,g'(x)-f'(x)g(x)$$

72.(B) $I=\int\frac{\sin^2 x}{1+\sin^2 x}dx=\int\left(1-\frac{1}{1+\sin^2 x}\right)dx=\int\left(1-\frac{\sec^2 x}{1+2\tan^2 x}\right)dx=x-\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+C$

73.(AD) $\int\frac{1}{\left(x^2+1\right)\left(x^2+4\right)}dx=\frac{1}{3}\int\left(\frac{1}{x^2+1}-\frac{1}{x^2+4}\right)dx=\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\frac{x}{2}+C$

Therefore, $A=\frac{1}{3}$ and $B=-\frac{1}{6}$

74.(C) $F(x)=\int\frac{dx}{\cos^2 x\sqrt{1+\tan x}}=\int\frac{\sec^2 x\,dx}{\sqrt{1+\tan x}}$

Put $\tan x=t$ to get: $F(x)=\int\frac{dt}{\sqrt{1+t}}=2\sqrt{1+t}+C=2\sqrt{1+\tan x}+C$

$F(0)=4 \quad \Rightarrow C=2$

75.(A) $F(x)=\int\frac{dx}{4-3\cos^2 x+5\sin^2 x}=\int\frac{\sec^2 x\,dx}{4\left(1+\tan^2 x\right)-3+5\tan^2 x}$

Put $\tan x=t$ to get $F(x)=\int\frac{dt}{9t^2+1}+C=\frac{1}{3}\tan^{-1}(3t)+C=\frac{1}{3}\tan^{-1}(3\tan x)+C$

76.(D) $I=\int\frac{dx}{x^4+5x^2+4}=\int\frac{dx}{\left(x^2+1\right)\left(x^2+4\right)}=\int\frac{1}{3}\left[\frac{1}{x^2+1}-\frac{1}{x^2+4}\right]dx \quad \Rightarrow \quad I=\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)+C$